DATA STRUCTURES USING C

Revision 1.1

1 December, 2016

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2016-2017

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UNIT-1
Introduction to Data Structures

Data Structure is a way of collecting and organising data in such a way that we can perform operations on these data in an effective way. Data Structures is about rendering data elements in terms of some relationship, for better organization and storage. For example, we have data player's name "Virat" and age 26. Here "Virat" is of **String** data type and 26 is of **integer** data type.

We can organize this data as a record like **Player** record. Now we can collect and store player's records in a file or database as a data structure. For example: "Dhoni" 30, "Gambhir" 31, "Sehwag" 33

In simple language, Data Structures are structures programmed to store ordered data, so that various operations can be performed on it easily. It represents the knowledge of data to be organized in memory. It should be designed and implemented in such a way that it reduces the complexity and increases the effieciency.

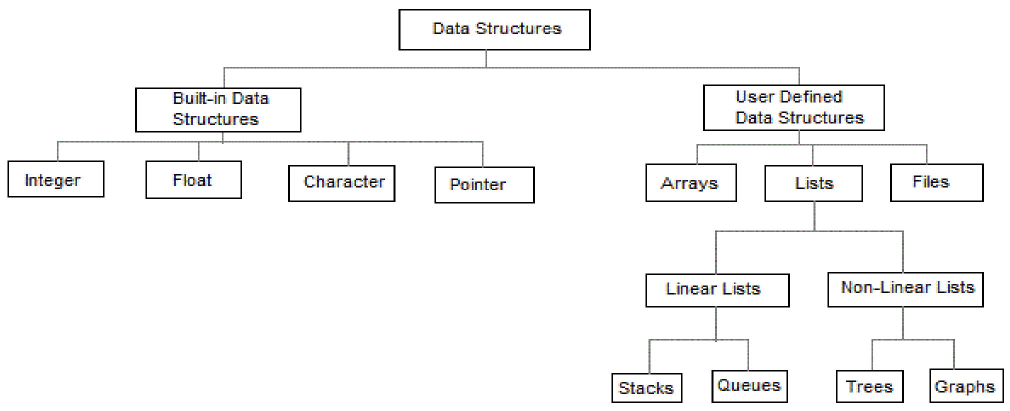
Basic types of Data Structures

As we have discussed above, anything that can store data can be called as a data structure, hence Integer, Float, Boolean, Char etc, all are data structures. They are known as **Primitive Data Structures**.

Then we also have some complex Data Structures, which are used to store large and connected data. Some example of **Abstract Data Structure** are :

- Linked List
- Tree
- Graph
- Stack, Queue etc.

All these data structures allow us to perform different operations on data. We select these data structures based on which type of operation is required. We will look into these data structures in more details in our later lessons.



INTRODUCTION TO DATA STRUCTURES

The data structures can also be classified on the basis of the following characteristics:

Characterstic	**Description**
Linear	In Linear data structures,the data items are arranged in a linear sequence. Example: **Array**
Non-Linear	In Non-Linear data structures,the data items are not in sequence. Example: **Tree**, **Graph**
Homogeneous	In homogeneous data structures,all the elements are of same type. Example: **Array**
Non-Homogeneous	In Non-Homogeneous data structure, the elements may or may not be of the same type. Example: **Structures**
Static	Static data structures are those whose sizes and structures associated memory locations are fixed, at compile time. Example: **Array**
Dynamic	Dynamic structures are those which expands or shrinks depending upon the program need and its execution. Also, their associated memory locations changes. Example: **Linked List created using pointers**

What is an Algorithm ?

An algorithm is a finite set of instructions or logic, written in order, to accomplish a certain predefined task. Algorithm is not the complete code or program, it is just the core logic(solution) of a problem, which can be expressed either as an informal high level description as **pseudocode** or using a **flowchart**.

Every Algorithm must satisfy the following properties:

1. **Input**- There should be 0 or more inputs supplied externally to the algorithm.
2. **Output**- There should be atleast 1 output obtained.
3. **Definiteness**- Every step of the algorithm should be clear and well defined.
4. **Finiteness**- The algorithm should have finite number of steps.
5. **Correctness**- Every step of the algorithm must generate a correct output.

An algorithm is said to be efficient and fast, if it takes less time to execute and consumes less memory space. The performance of an algorithm is measured on the basis of following properties :

1. Time Complexity
2. Space Complexity

Space Complexity

Its the amount of memory space required by the algorithm, during the course of its execution. Space complexity must be taken seriously for multi-user systems and in situations where limited memory is available.

An algorithm generally requires space for following components :

- **Instruction Space :** Its the space required to store the executable version of the program. This space is fixed, but varies depending upon the number of lines of code in the program.
- **Data Space :** Its the space required to store all the constants and variables value.
- **Environment Space :** Its the space required to store the environment information needed to resume the suspended function.

Time Complexity

Time Complexity is a way to represent the amount of time needed by the program to run till its completion. We will study this in details in later sections.

Time Complexity of Algorithms

Time complexity of an algorithm signifies the total time required by the program to run till its completion. The time complexity of algorithms is most commonly expressed using the **big O notation**.

Time Complexity is most commonly estimated by counting the number of elementary functions performed by the algorithm. And since the algorithm's performance may vary with different types of input data, hence for an algorithm we usually use the **worst-case Time complexity** of an algorithm because that is the maximum time taken for any input size.

Calculating Time Complexity

Now lets tap onto the next big topic related to Time complexity, which is How to Calculate Time Complexity. It becomes very confusing some times, but we will try to explain it in the simplest way.

Now the most common metric for calculating time complexity is Big O notation. This removes all constant factors so that the running time can be estimated in relation to **N**, as N approaches infinity. In general you can think of it like this :

```
statement;
```

Above we have a single statement. Its Time Complexity will be **Constant**. The running time of the statement will not change in relation to N.

```
for(i=0; i < N; i++)

{

  statement;

}
```

The time complexity for the above algorithm will be **Linear**. The running time of the loop is directly proportional to N. When N doubles, so does the running time.

```
for(i=0; i < N; i++)
{
  for(j=0; j < N;j++)
  {
    statement;
  }
}
```

This time, the time complexity for the above code will be **Quadratic**. The running time of the two loops is proportional to the square of N. When N doubles, the running time increases by N * N.

```
while(low <= high)
{
  mid = (low + high) / 2;
  if (target < list[mid])
    high = mid - 1;
  else if (target > list[mid])
    low = mid + 1;
  else break;
}
```

This is an algorithm to break a set of numbers into halves, to search a particular field(we will study this in detail later). Now, this algorithm will have a **Logarithmic** Time Complexity. The running time of the algorithm is proportional to the number of times N can be divided by 2(N is high-low here). This is because the algorithm divides the working area in half with each iteration.

```
void quicksort(int list[], int left, int right)
{
  int pivot = partition(list, left, right);
  quicksort(list, left, pivot - 1);
  quicksort(list, pivot + 1, right);
}
```

Taking the previous algorithm forward, above we have a small logic of Quick Sort(we will study this in detail later). Now in Quick Sort, we divide the list into halves every time, but we repeat the iteration N times(where N is the size of list). Hence time complexity will be **N*log(N)**. The running time consists of N loops (iterative or recursive) that are logarithmic, thus the algorithm is a combination of linear and logarithmic.

NOTE : In general, doing something with every item in one dimension is linear, doing something with every item in two dimensions is quadratic, and dividing the working area in half is logarithmic.

Types of Notations for Time Complexity

Now we will discuss and understand the various notations used for Time Complexity.

1. **Big Oh** denotes "*fewer than or the same as*" <expression> iterations.
2. **Big Omega** denotes "*more than or the same as*" <expression> iterations.
3. **Big Theta** denotes "*the same as*" <expression> iterations.
4. **Little Oh** denotes "*fewer than*" <expression> iterations.
5. **Little Omega** denotes "*more than*" <expression> iterations.

Understanding Notations of Time Complexity with Example

O(expression) is the set of functions that grow slower than or at the same rate as expression. It indicates the maximum required by an algorithm for all input values. It represents the worst case of an algorithm's time complexity.

Omega(expression) is the set of functions that grow faster than or at the same rate as expression. It indicates the minimum time required by an algorithm for all input values. It represents the best case of an algorithm's time complexity.

Theta(expression) consist of all the functions that lie in both O(expression) and Omega(expression). It indicates the average bound of an algorithm. It represents the average case of an algorithm's time complexity.

Suppose you've calculated that an algorithm takes f(n) operations, where,

```
f(n) = 3*n^2 + 2*n + 4.   // n^2 means square of n
```

Since this polynomial grows at the same rate as **n^2**, then you could say that the function **f** lies in the set **Theta(n^2)**. (It also lies in the sets **O(n^2)** and **Omega(n^2)** for the same reason.)

The simplest explanation is, because **Theta** denotes *the same as* the expression. Hence, as **f(n)**grows by a factor of **n^2**, the time complexity can be best represented as **Theta(n^2)**.

UNIT-2
Introduction to Sorting

Sorting is nothing but storage of data in sorted order, it can be in ascending or descending order. The term Sorting comes into picture with the term Searching. There are so many things in our real life that we need to search, like a particular record in database, roll numbers in merit list, a particular telephone number, any particular page in a book etc.

Sorting arranges data in a sequence which makes searching easier. Every record which is going to be sorted will contain one key. Based on the key the record will be sorted. For example, suppose we have a record of students, every such record will have the following data:

- Roll No.
- Name
- Age
- Class

Here Student roll no. can be taken as key for sorting the records in ascending or descending order. Now suppose we have to search a Student with roll no. 15, we don't need to search the complete record we will simply search between the Students with roll no. 10 to 20.

Sorting Efficiency

There are many techniques for sorting. Implementation of particular sorting technique depends upon situation. Sorting techniques mainly depends on two parameters. First parameter is the execution time of program, which means time taken for execution of program. Second is the space, which means space taken by the program.

Types of Sorting Techniques

There are many types of Sorting techniques, differentiated by their efficiency and space requirements. Following are some sorting techniques which we will be covering in next sections.

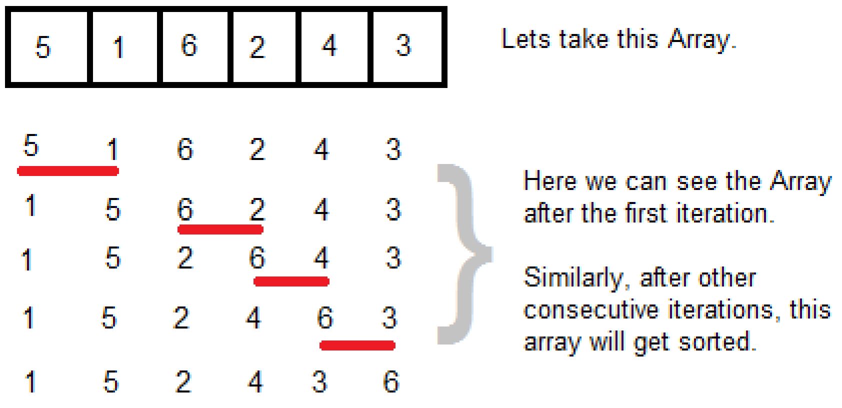
1. Bubble Sort
2. Insertion Sort
3. Selection Sort
4. Quick Sort
5. Merge Sort
6. Heap Sort

Bubble Sorting

Bubble Sort is an algorithm which is used to sort **N** elements that are given in a memory for eg: an Array with **N** number of elements. Bubble Sort compares all the element one by one and sort them based on their values.

It is called Bubble sort, because with each iteration the largest element in the list bubbles up towards the last place, just like a water bubble rises up to the water surface.

Sorting takes place by stepping through all the data items one-by-one in pairs and comparing adjacent data items and swapping each pair that is out of order.


5 1 6 2 4 3
Lets take this Array.
5 1 6 2 4 3
1 5 6 2 4 3
1 5 2 6 4 3
1 5 2 4 6 3
1 5 2 4 3 6
Here we can see the Array after the first iteration.
Similarly, after other consecutive iterations, this array will get sorted.

Sorting using Bubble Sort Algorithm

Let's consider an array with values {5, 1, 6, 2, 4, 3}

```
int a[6] = {5, 1, 6, 2, 4, 3};
int i, j, temp;
for(i=0; i<6; i++)
{
  for(j=0; j<6-i-1; j++)
  {
    if( a[j] > a[j+1])
    {
      temp = a[j];
      a[j] = a[j+1];
      a[j+1] = temp;
    }
  }
}
//now you can print the sorted array after this
```

Above is the algorithm, to sort an array using Bubble Sort. Although the above logic will sort and unsorted array, still the above algorithm is not efficient because as per the above logic, the for-loop will keep executing for six iterations even if the array gets sorted after the second iteration.

Hence we can insert a flag and can keep checking whether swapping of elements is taking place or not in the following iteration. If no swapping is taking place, it means the array is sorted and we can jump out of the for loop, instead executing all the iterations.

```
int a[6] = {5, 1, 6, 2, 4, 3};
int i, j, temp;
for(i=0; i<6; i++)
```

```
{
  int flag = 0;        //taking a flag variable
  for(j=0; j<6-i-1; j++)
  {
    if( a[j] > a[j+1])
    {
      temp = a[j];
      a[j] = a[j+1];
      a[j+1] = temp;
      flag = 1;         //setting flag as 1, if swapping occurs
    }
  }
  if(!flag)             //breaking out of for loop if no swapping takes place
  {
    break;
  }
}
```

In the above code, if in a complete single cycle of j iteration(inner for loop), if no swapping takes place, then flag will remain 0 and then we will break out of the for loops, because the array has already been sorted.

Complexity Analysis of Bubble Sorting

In Bubble Sort, n-1 comparisons will be done in 1st pass, n-2 in 2nd pass, n-3 in 3rd pass and so on. So the total number of comparisons will be

```
(n-1)+(n-2)+(n-3)+.....+3+2+1
Sum = n(n-1)/2
i.e O(n^2)
Hence the complexity of Bubble Sort is O(n^2).
```

The main advantage of Bubble Sort is the simplicity of the algorithm. Space complexity for Bubble Sort is **O(1)**, because only single additional memory space is required i.e. for **temp** variable

Best-case Time Complexity will be **O(n)**, it is when the list is already sorted.

Complexity Analysis of Bubble Sort

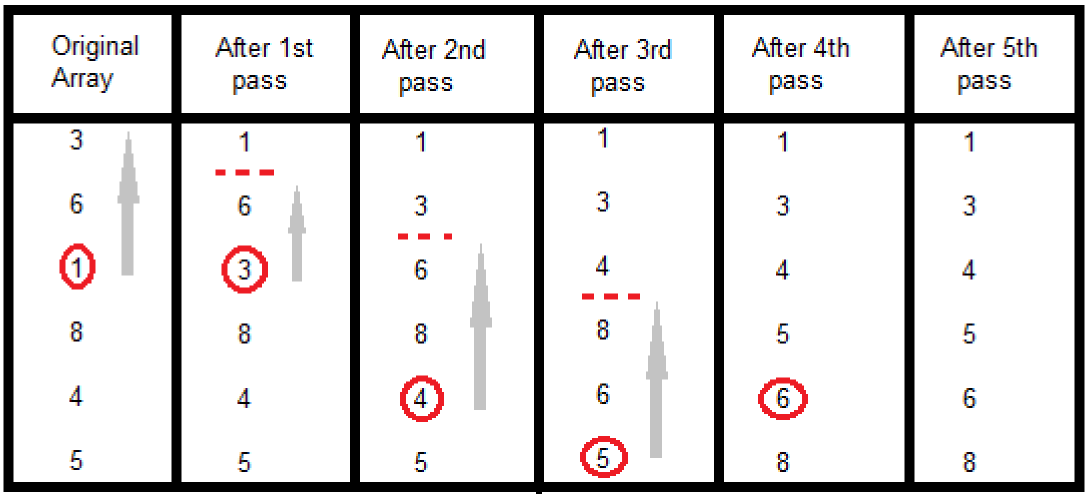
```
Worst Case Time Complexity : O(n^2)
Best Case Time Complexity : O(n)
Average Time Complexity : O(n^2)
Space Complexity : O(1)
```

Selection Sorting

Selection sorting is conceptually the most simplest sorting algorithm. This algorithm first finds the smallest element in the array and exchanges it with the element in the first position, then find the second smallest element and exchange it with the element in the second position, and continues in this way until the entire array is sorted.

Note: Selection sort is an unstable sort i.e it might change the occurrence of two similar elements in the list while sorting. But it can also be a stable sort when it is implemented using linked list data structures.

How Selection Sorting Works



Original Array	After 1st pass	After 2nd pass	After 3rd pass	After 4th pass	After 5th pass
3	1	1	1	1	1
6	6	3	3	3	3
1	3	6	4	4	4
8	8	8	8	5	5
4	4	4	6	6	6
5	5	5	5	8	8

In the first pass, the smallest element found is 1, so it is placed at the first position, then leaving the first element, next smallest element is searched from the rest of the elements. We get 3 as the smallest, so it is then placed at the second position. Then leaving 1 and 3, we search for the next smallest element from the rest of the elements and put it at third position and keep doing this until array is sorted.

Sorting using Selection Sort Algorithm

```
void selectionSort(int a[], int size)
{
  int i, j, min, temp;
  for(i=0; i < size-1; i++ )
  {
    min = i;   //setting min as i
    for(j=i+1; j < size; j++)
    {
      if(a[j] < a[min])   //if element at j is less than element at min position
      {
       min = j;    //then set min as j
      }
    }
   temp = a[i];
   a[i] = a[min];
   a[min] = temp;
  }
}
```

Complexity Analysis of Selection Sorting

```
Worst Case Time Complexity : O(n^2)
Best Case Time Complexity : O(n^2)
Average Time Complexity : O(n^2)
Space Complexity : O(1)
```

Quick Sort Algorithm

Quick Sort, as the name suggests, sorts any list very quickly. Quick sort is not a stable search, but it is very fast and requires very less additional space. It is based on the rule of **Divide and Conquer**(also called *partition-exchange sort*). This algorithm divides the list into three main parts :

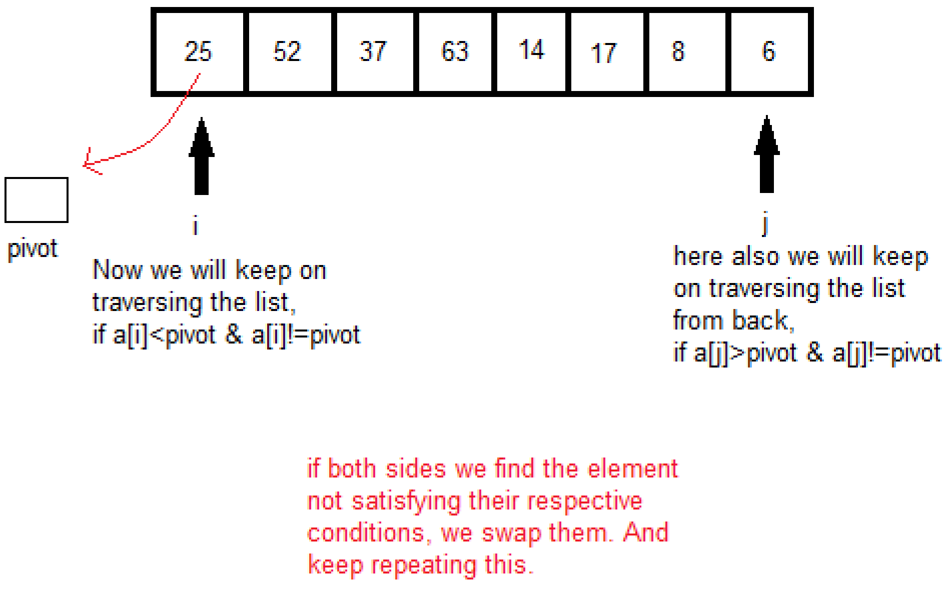
1. Elements less than the Pivot element
2. Pivot element(Central element)
3. Elements greater than the pivot element

In the list of elements, mentioned in below example, we have taken **25** as pivot. So after the first pass, the list will be changed like this.

`6 8 17 14 25 63 37 52`

Hence after the first pass, pivot will be set at its position, with all the elements smaller to it on its left and all the elements larger than to its right. Now `6 8 17 14` and `63 37 52` are considered as two separate lists, and same logic is applied on them, and we keep doing this until the complete list is sorted.

How Quick Sorting Works


25
52
37
63
14
17
8
6
pivot
i
Now we will keep on traversing the list, if a[i]<pivot & a[i]!=pivot
j
here also we will keep on traversing the list from back, if a[j]>pivot & a[j]!=pivot
if both sides we find the element not satisfying their respective conditions, we swap them. And keep repeating this.

DIVIDE AND CONQUER - QUICK SORT

Sorting using Quick Sort Algorithm

```
/*  a[] is the array, p is starting index, that is 0,
and r is the last index of array.  */

void quicksort(int a[], int p, int r)
{
  if(p < r)
  {
    int q;
    q = partition(a, p, r);
    quicksort(a, p, q);
    quicksort(a, q+1, r);
  }
}

int partition(int a[], int p, int r)
{
  int i, j, pivot, temp;
  pivot = a[p];
  i = p;
  j = r;
  while(1)
  {
   while(a[i] < pivot && a[i] != pivot)
   i++;
   while(a[j] > pivot && a[j] != pivot)
   j--;
   if(i < j)
   {
    temp = a[i];
    a[i] = a[j];
    a[j] = temp;
   }
   else
   {
    return j;
   }
  }
}
```

Complexity Analysis of Quick Sort

Worst Case Time Complexity : O(n^2)

Best Case Time Complexity : O(n log n)

Average Time Complexity : O(n log n)

Space Complexity : O(n log n)

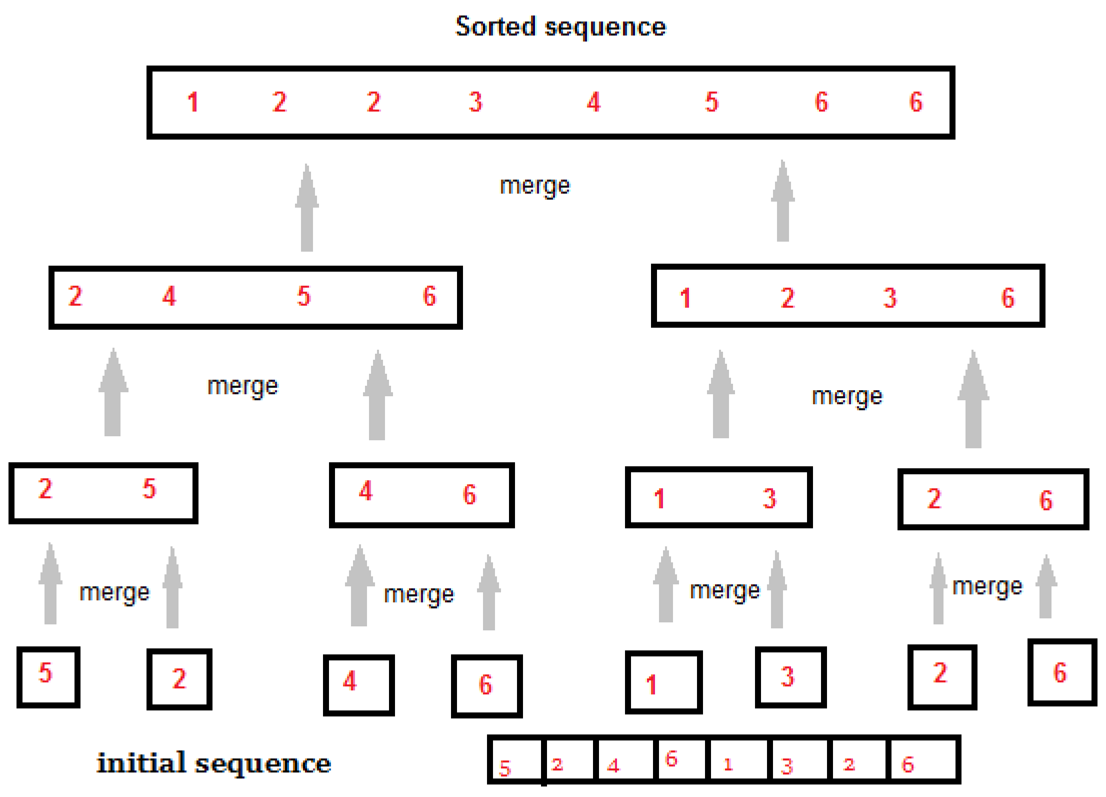
- Space required by quick sort is very less, only O(n log n) additional space is required.
- Quick sort is not a stable sorting technique, so it might change the occurence of two similar elements in the list while sorting.

Merge Sort Algorithm

Merge Sort follows the rule of **Divide and Conquer**. In merge sort the unsorted list is divided into N sublists, each having one element, because a list consisting of one element is always sorted. Then, it repeatedly merges these sublists, to produce new sorted sublists, and in the end, only one sorted list is produced.

Merge Sort is quite fast, and has a time complexity of **O(n log n)**. It is also a stable sort, which means the "equal" elements are ordered in the same order in the sorted list.

How Merge Sort Works



Like we can see in the above example, merge sort first breaks the unsorted list into sorted sublists, each having one element, because a list of one element is considered sorted and then it keeps merging these sublists, to finally get the complete sorted list.

Sorting using Merge Sort Algorithm

```
/*  a[] is the array, p is starting index, that is 0,
and r is the last index of array.  */

//Lets take a[5] = {32, 45, 67, 2, 7} as the array to be sorted.

void mergesort(int a[], int p, int r)
{
  int q;
  if(p < r)
  {
    q = floor( (p+r) / 2);
    mergesort(a, p, q);
    mergesort(a, q+1, r);
    merge(a, p, q, r);
  }
}

void merge(int a[], int p, int q, int r)
{
  int b[5];     //same size of a[]
  int i, j, k;
  k = 0;
  i = p;
  j = q+1;
  while(i <= q && j <= r)
  {
    if(a[i] < a[j])
    {
      b[k++] = a[i++];       // same as b[k]=a[i]; k++; i++;
    }
    else
    {
      b[k++] = a[j++];
    }
  }

  while(i <= q)
  {
    b[k++] = a[i++];
  }

  while(j <= r)
  {
    b[k++] = a[j++];
  }

  for(i=r; i >= p; i--)
  {
    a[i] = b[--k];        // copying back the sorted list to a[]
  }

}
```

Complexity Analysis of Merge Sort

```
Worst Case Time Complexity : O(n log n)

Best Case Time Complexity : O(n log n)

Average Time Complexity : O(n log n)

Space Complexity : O(n)
```

- Time complexity of Merge Sort is O(n Log n) in all 3 cases (worst, average and best) as merge sort always divides the array in two halves and take linear time to merge two halves.
- It requires equal amount of additional space as the unsorted list. Hence its not at all recommended for searching large unsorted lists.
- It is the best Sorting technique used for sorting **Linked Lists**.

Heap Sort Algorithm

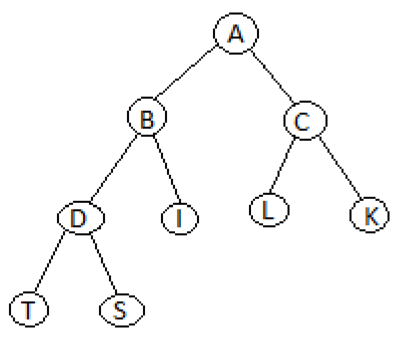
Heap Sort is one of the best sorting methods being in-place and with no quadratic worst-case scenarios. Heap sort algorithm is divided into two basic parts :

- Creating a Heap of the unsorted list.
- Then a sorted array is created by repeatedly removing the largest/smallest element from the heap, and inserting it into the array. The heap is reconstructed after each removal.

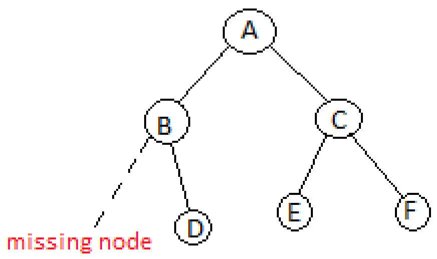
What is a Heap ?

Heap is a special tree-based data structure, that satisfies the following special heap properties:

1. **Shape Property :** Heap data structure is always a Complete Binary Tree, which means all levels of the tree are fully filled.

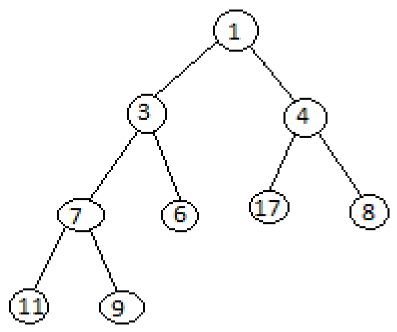


Complete Binary Tree



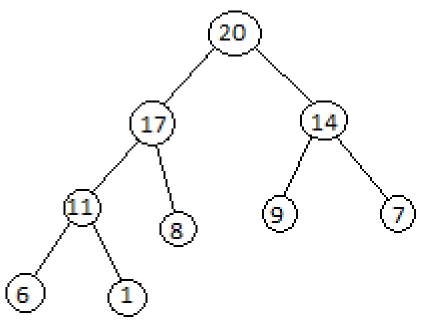
In-Complete Binary Tree

2. **Heap Property :** All nodes are either *[greater than or equal to]* or *[less than or equal to]* each of its children. If the parent nodes are greater than their child nodes, heap is called a **Max-Heap**, and if the parent nodes are smaller than their child nodes, heap is called **Min-Heap**.



Min-Heap

In min-heap, first element is the smallest. So when we want to sort a list in ascending order, we create a Min-heap from that list, and picks the first element, as it is the smallest, then we repeat the process with remaining elements.



Max-Heap

In max-heap, the first element is the largest, hence it is used when we need to sort a list in descending order.

How Heap Sort Works

Initially on receiving an unsorted list, the first step in heap sort is to create a Heap data structure(Max-Heap or Min-Heap). Once heap is built, the first element of the Heap is either largest or smallest(depending upon Max-Heap or Min-Heap), so we put the first element of the heap in our array. Then we again make heap using the remaining elements, to again pick the first element of the heap and put it into the array. We keep on doing the same repeatedly untill we have the complete sorted list in our array.

In the below algorithm, initially **heapsort()** function is called, which calls **buildheap()** to build heap, which inturn uses **satisfyheap()** to build the heap.

Sorting using Heap Sort Algorithm

```
/*  Below program is written in C++ language  */

void heapsort(int[], int);
void buildheap(int [], int);
void satisfyheap(int [], int, int);

void main()
{
  int a[10], i, size;
  cout << "Enter size of list";    // less than 10, because max size of array is 10
  cin >> size;
  cout << "Enter" << size << "elements";
  for( i=0; i < size; i++)
  {
    cin >> a[i];
  }
  heapsort(a, size);
  getch();
}

void heapsort(int a[], int length)
{
  buildheap(a, length);
  int heapsize, i, temp;
  heapsize = length - 1;
  for( i=heapsize; i >= 0; i--)
  {
    temp = a[0];
    a[0] = a[heapsize];
    a[heapsize] = temp;
    heapsize--;
    satisfyheap(a, 0, heapsize);
  }
  for( i=0; i < length; i++)
  {
    cout << "\t" << a[i];
  }
}

void buildheap(int a[], int length)
{
  int i, heapsize;
  heapsize = length - 1;
  for( i=(length/2); i >= 0; i--)
  {
    satisfyheap(a, i, heapsize);
  }
}

void satisfyheap(int a[], int i, int heapsize)
{
  int l, r, largest, temp;
  l = 2*i;
  r = 2*i + 1;
  if(l <= heapsize && a[l] > a[i])
  {
    largest = l;
  }
  else
  {
    largest = i;
  }
  if( r <= heapsize && a[r] > a[largest])
  {
    largest = r;
  }
  if(largest != i)
  {
    temp = a[i];
    a[i] = a[largest];
    a[largest] = temp;
    satisfyheap(a, largest, heapsize);
  }
}
```

Complexity Analysis of Heap Sort

```
Worst Case Time Complexity : O(n log n)

Best Case Time Complexity : O(n log n)

Average Time Complexity : O(n log n)

Space Complexity : O(1)
```

- Heap sort is not a Stable sort, and requires a constant space for sorting a list.
- Heap Sort is very fast and is widely used for sorting.

Searching Algorithms on Array

Before studying searching algorithms on array we should know what is an algorithm?

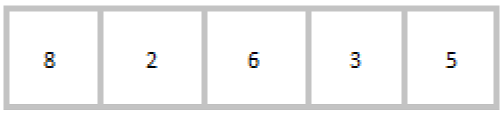
An **algorithm** is a step-by-step procedure or method for solving a problem by a computer in a given number of steps. The steps of an algorithm may include repetition depending upon the problem for which the algorithm is being developed. The algorithm is written in human readable and understandable form. To search an element in a given array, it can be done in two ways: Linear search and Binary search.

Linear Search

A linear search is the basic and simple search algorithm. A linear search searches an element or value from an array till the desired element or value is not found and it searches in a sequence order. It compares the element with all the other elements given in the list and if the element is matched it returns the value index else it return -1. Linear Search is applied on the unsorted or unordered list when there are fewer elements in a list.

Example with Implementation

To search the element 5 it will go step by step in a sequence order.



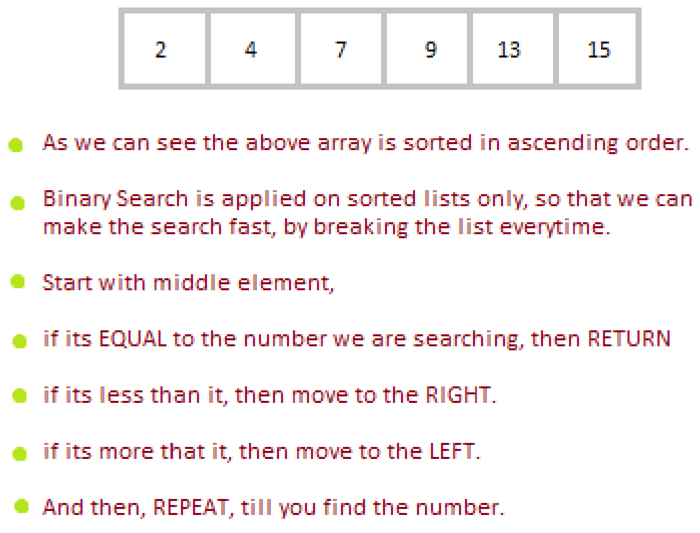
```
function findIndex(values, target)
 {
   for(var i = 0; i < values.length; ++i)
     {
       if (values[i] == target)
         {
           return i;
         }
     }
   return -1;
 }
//call the function findIndex with array and number to be searched
findIndex([ 8 , 2 , 6 , 3 , 5 ] , 5) ;
```

Binary Search

Binary Search is applied on the sorted array or list. In binary search, we first compare the value with the elements in the middle position of the array. If the value is matched, then we return the value. If the value is less than the middle element, then it must lie in the lower half of the array and if it's greater than the element then it must lie in the upper half of the array. We repeat this procedure on the lower (or upper) half of the array. Binary Search is useful when there are large numbers of elements in an array.

Example with Implementation

To search an element 13 from the sorted array or list.



- As we can see the above array is sorted in ascending order.
- Binary Search is applied on sorted lists only, so that we can make the search fast, by breaking the list everytime.
- Start with middle element,
- if its EQUAL to the number we are searching, then RETURN
- if its less than it, then move to the RIGHT.
- if its more that it, then move to the LEFT.
- And then, REPEAT, till you find the number.

```
function findIndex(values, target)

{

  return binarySearch(values, target, 0, values.length - 1);

};


function binarySearch(values, target, start, end) {

  if (start > end) { return -1; } //does not exist
  var middle = Math.floor((start + end) / 2);
  var value = values[middle];

  if (value > target) { return binarySearch(values, target, start, middle-1); }
  if (value < target) { return binarySearch(values, target, middle+1, end); }
  return middle; //found!
}

findIndex([2, 4, 7, 9, 13, 15], 13);
```

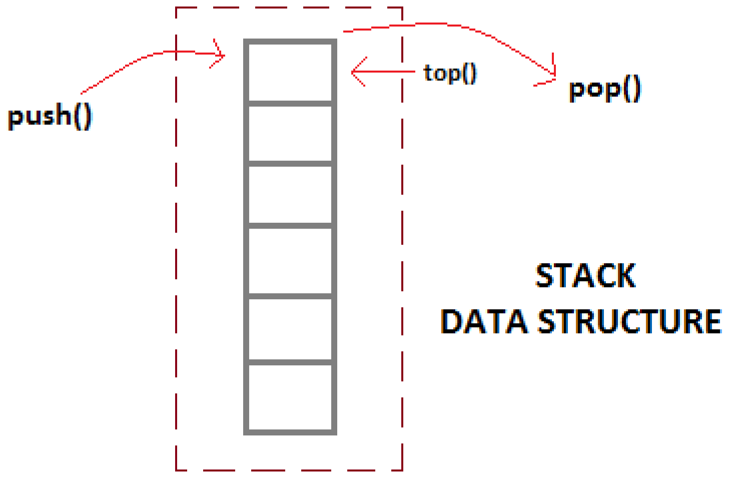
In the above program logic, we are first comparing the middle number of the list, with the target, if it matches we return. If it doesn't, we see whether the middle number is greater than or smaller than the target.

If the Middle number is greater than the Target, we start the binary search again, but this time on the left half of the list, that is from the start of the list to the middle, not beyond that.

If the Middle number is smaller than the Target, we start the binary search again, but on the right half of the list, that is from the middle of the list to the end of the list.

UNIT - 3
Stacks

Stack is an abstract data type with a bounded(predefined) capacity. It is a simple data structure that allows adding and removing elements in a particular order. Every time an element is added, it goes on the top of the stack, the only element that can be removed is the element that was at the top of the stack, just like a pile of objects.



Basic features of Stack

1. Stack is an ordered list of similar data type.
2. Stack is a **LIFO** structure. (Last in First out).
3. **push()** function is used to insert new elements into the Stack and **pop()** function is used to delete an element from the stack. Both insertion and deletion are allowed at only one end of Stack called **Top**.
4. Stack is said to be in **Overflow** state when it is completely full and is said to be in **Underflow** state if it is completely empty.

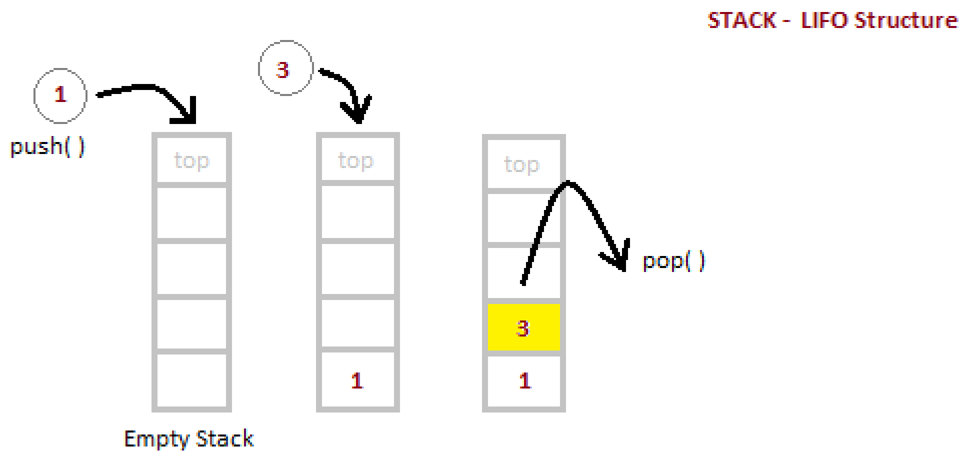
Applications of Stack

The simplest application of a stack is to reverse a word. You push a given word to stack - letter by letter - and then pop letters from the stack.

There are other uses also like : **Parsing**, **Expression Conversion**(Infix to Postfix, Postfix to Prefix etc) and many more.

Implementation of Stack

Stack can be easily implemented using an Array or a Linked List. Arrays are quick, but are limited in size and Linked List requires overhead to allocate, link, unlink, and deallocate, but is not limited in size. Here we will implement Stack using array.



In a Stack, all operations take place at the "**top**" of the stack. The "**push**" operation adds an item to the top of the Stack.
The "**pop**" operation removes the item on top of the stack.

Algorithm for PUSH operation

1. Check if the stack is full or not.
2. If the stack is full,then print error of overflow and exit the program.
3. If the stack is not full,then increment the top and add the element .

Algorithm for POP operation

1. Check if the stack is empty or not.
2. If the stack is empty, then print error of underflow and exit the program.
3. If the stack is not empty, then print the element at the top and decrement the top.

```
/*  Below program is written in C++ language  */

Class Stack
{
  int top;
  public:
  int a[10];    //Maximum size of Stack
  Stack()
  {
    top = -1;
  }
};

void Stack::push(int x)
{
  if( top >= 10)
  {
    cout << "Stack Overflow";
  }
  else
  {
    a[++top] = x;
    cout << "Element Inserted";
  }
}

int Stack::pop()
{
  if(top < 0)
  {
    cout << "Stack Underflow";
    return 0;
  }
  else
  {
    int d = a[top--];
    return d;
  }
}

void Stack::isEmpty()
{
  if(top < 0)
  {
    cout << "Stack is empty";
  }
  else
  {
    cout << "Stack is not empty";
  }
}
```

Position of Top	**Status of Stack**
-1	Stack is Empty
0	Only one element in Stack
N-1	Stack is Full
N	Overflow state of Stack

Analysis of Stacks

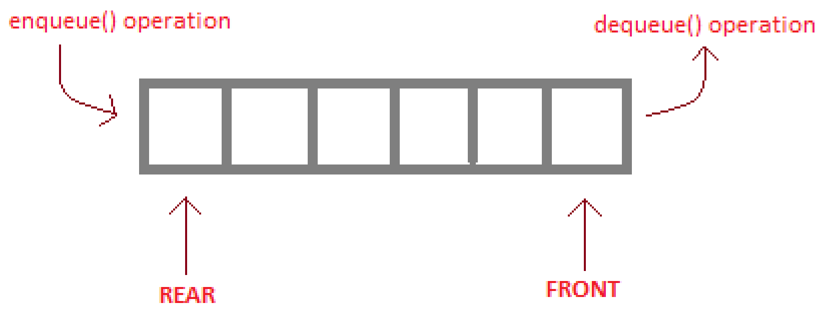
Below mentioned are the time complexities for various operations that can be performed on the Stack data structure.

- **Push Operation** : O(1)
- **Pop Operation** : O(1)
- **Top Operation** : O(1)
- **Search Operation** : O(n)

Queue Data Structures

Queue is also an abstract data type or a linear data structure, in which the first element is inserted from one end called **REAR**(also called tail), and the deletion of existing element takes place from the other end called as **FRONT**(also called head). This makes queue as FIFO(First in First Out) data structure, which means that element inserted first will also be removed first.

The process to add an element into queue is called **Enqueue** and the process of removal of an element from queue is called **Dequeue**.



enqueue( ) is the operation for adding an element into Queue.

dequeue( ) is the operation for removing an element from Queue .

QUEUE DATA STRUCTURE

Basic features of Queue

1. Like Stack, Queue is also an ordered list of elements of similar data types.
2. Queue is a FIFO(First in First Out) structure.
3. Once a new element is inserted into the Queue, all the elements inserted before the new element in the queue must be removed, to remove the new element.
4. **peek( )** function is oftenly used to return the value of first element without dequeuing it.

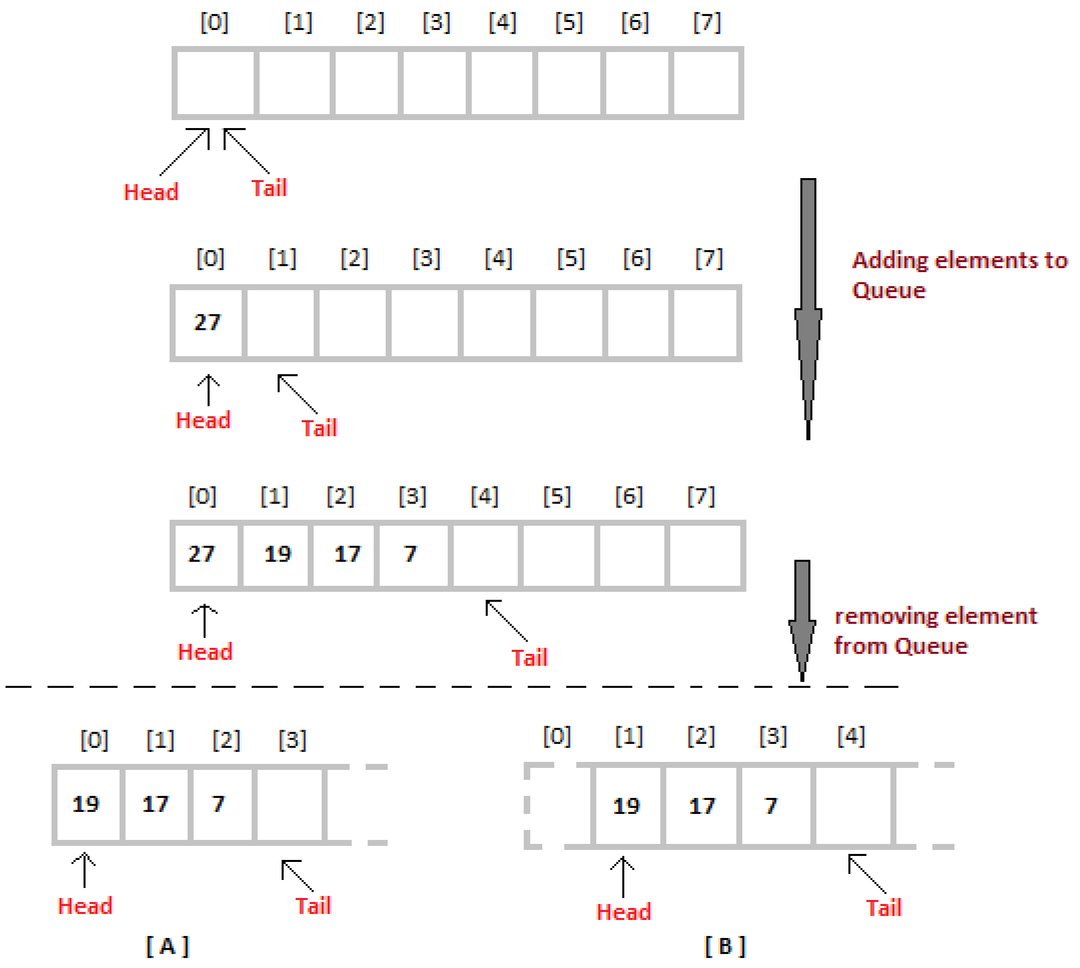
Applications of Queue

Queue, as the name suggests is used whenever we need to manage any group of objects in an order in which the first one coming in, also gets out first while the others wait for their turn, like in the following scenarios :

1. Serving requests on a single shared resource, like a printer, CPU task scheduling etc.
2. In real life scenario, Call Center phone systems uses Queues to hold people calling them in an order, until a service representative is free.
3. Handling of interrupts in real-time systems. The interrupts are handled in the same order as they arrive i.e First come first served.

Implementation of Queue

Queue can be implemented using an Array, Stack or Linked List. The easiest way of implementing a queue is by using an Array. Initially the **head**(FRONT) and the **tail**(REAR) of the queue points at the first index of the array (starting the index of array from 0). As we add elements to the queue, the tail keeps on moving ahead, always pointing to the position where the next element will be inserted, while the head remains at the first index.



When we remove an element from Queue, we can follow two possible approaches (mentioned [A] and [B] in above diagram). In [A] approach, we remove the element at **head** position, and then one by one shift all the other elements in forward position. In approach [B] we remove the element from **head**position and then move **head** to the next position.

In approach [A] there is an overhead of shifting the elements one position forward every time we remove the first element. In approach [B] there is no such overhead, but whenever we move head one position ahead, after removal of first element, the size on Queue is reduced by one space each time.

Algorithm for ENQUEUE operation

1. Check if the queue is full or not.
2. If the queue is full, then print overflow error and exit the program.
3. If the queue is not full, then increment the tail and add the element.

Algorithm for DEQUEUE operation

1. Check if the queue is empty or not.
2. If the queue is empty, then print underflow error and exit the program.
3. If the queue is not empty, then print the element at the head and increment the head.

```
/* Below program is written in C++ language */

#define SIZE 100
class Queue
{
  int a[100];
  int rear;     //same as tail
  int front;    //same as head

  public:
  Queue()
  {
    rear = front = -1;
  }
  void enqueue(int x);     //declaring enqueue, dequeue and display functions
  int dequeue();
  void display();
}

void Queue :: enqueue(int x)
{
  if( rear = SIZE-1)
  {
    cout << "Queue is full";
  }
  else
  {
    a[++rear] = x;
  }
}

int queue :: dequeue()
{
  return a[++front];     //following approach [B], explained above
}

void queue :: display()
{
  int i;
  for( i = front; i <= rear; i++)
  {
    cout << a[i];
  }
}
```

To implement approach [A], you simply need to change the dequeue method, and include a for loop which will shift all the remaining elements one position.

```
return a[0];      //returning first element

for (i = 0; i < tail-1; i++)      //shifting all other elements
```

```
{
  a[i]= a[i+1];
  tail--;
}
```

Analysis of Queue

- Enqueue : **O(1)**
- Dequeue : **O(1)**
- Size : **O(1)**

Queue Data Structure using Stack

A Queue is defined by its property of FIFO, which means First in First Out, i.e the element which is added first is taken out first. Hence we can implement a Queue using Stack for storage instead of array.

For performing **enqueue** we require only one stack as we can directly **push** data into stack, but to perform **dequeue** we will require two Stacks, because we need to follow queue's FIFO property and if we directly **pop** any data element out of Stack, it will follow LIFO approach(Last in First Out).

Implementation of Queue using Stacks

In all we will require two Stacks, we will call them InStack and OutStack.

```
class Queue {
  public:
  Stack S1, S2;
  //defining methods

  void enqueue(int x);

  int dequeue();
}
```

We know that, Stack is a data structure, in which data can be added using **push()** method and data can be deleted using **pop()** method. To learn about Stack, follow the link : Stack Data Structure

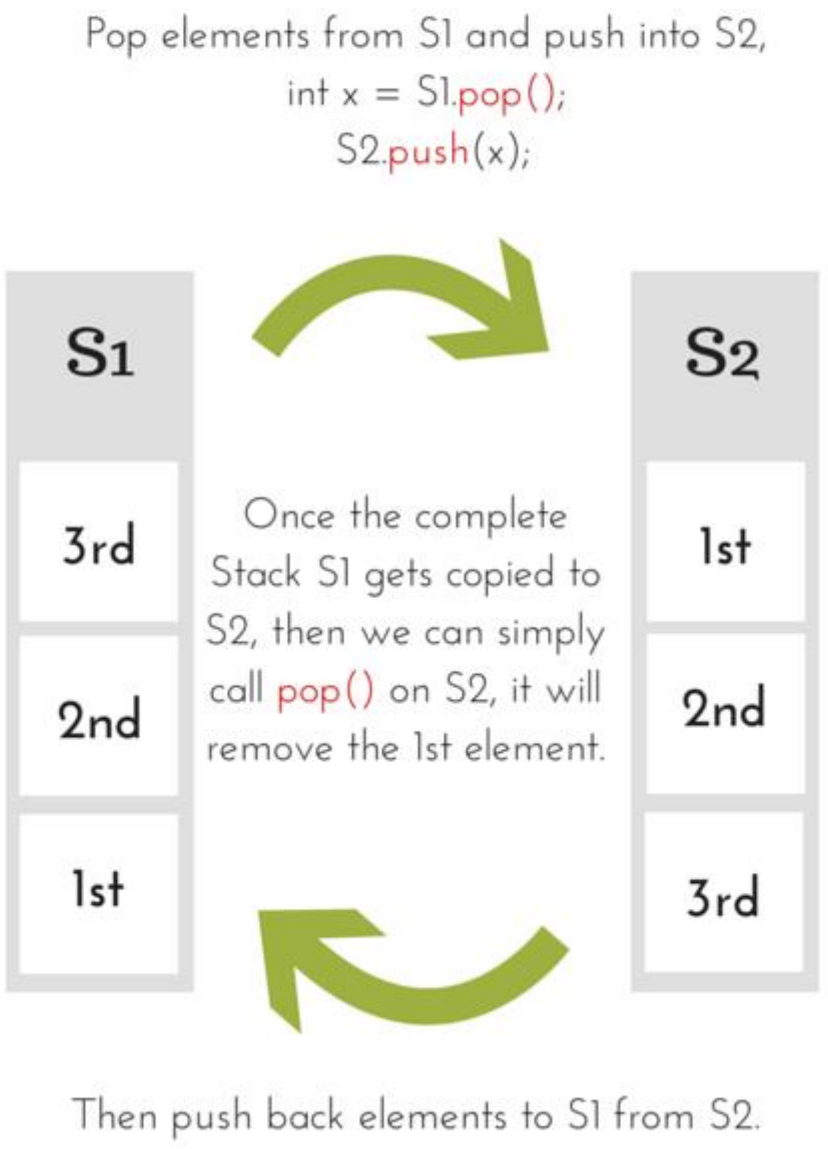
Adding Data to Queue

As our Queue has Stack for data storage in place of arrays, hence we will be adding data to Stack, which can be done using the `push()` method, hence :

```
void Queue :: enqueue(int x) {
  S1.push(x);
}
```

Removing Data from Queue

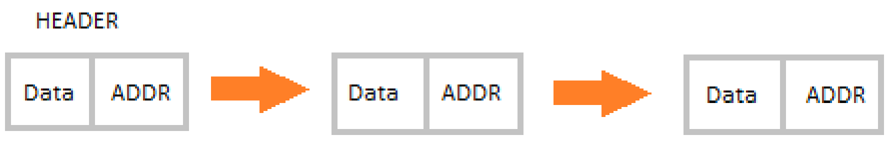
When we say remove data from Queue, it always means taking out the First element first and so on, as we have to follow the FIFO approach. But if we simply perform `S1.pop()` in our **dequeue** method, then it will remove the Last element first. So what to do now?


Pop elements from S1 and push into S2,
int x = S1.pop();
S2.push(x);
S1
3rd
2nd
1st
Once the complete Stack S1 gets copied to S2, then we can simply call pop() on S2, it will remove the 1st element.
S2
1st
2nd
3rd
Then push back elements to S1 from S2.

```
int Queue :: dequeue() {
  while(S1.isEmpty()) {
    x = S1.pop();
    S2.push();
  }

  //removing the element
  x = S2.pop();

  while(!S2.isEmpty()) {
    x = S2.pop();
    S1.push(x);
  }

  return x;
}
```

UNIT - 4
Introduction to Linked Lists

Linked List is a linear data structure and it is very common data structure which consists of group of nodes in a sequence which is divided in two parts. Each node consists of its own data and the address of the next node and forms a chain. Linked Lists are used to create trees and graphs.



Advantages of Linked Lists

- They are a dynamic in nature which allocates the memory when required.
- Insertion and deletion operations can be easily implemented.
- Stacks and queues can be easily executed.
- Linked List reduces the access time.

Disadvantages of Linked Lists

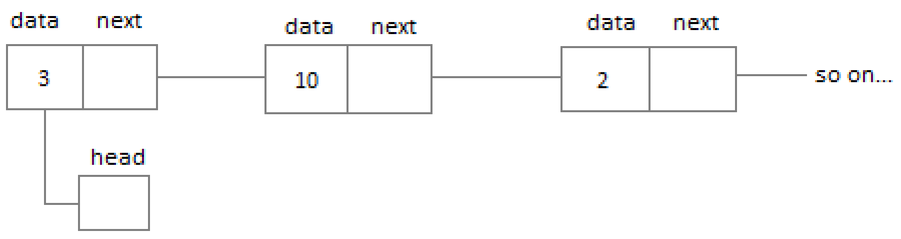
- The memory is wasted as pointers require extra memory for storage.
- No element can be accessed randomly; it has to access each node sequentially.
- Reverse Traversing is difficult in linked list.

Applications of Linked Lists

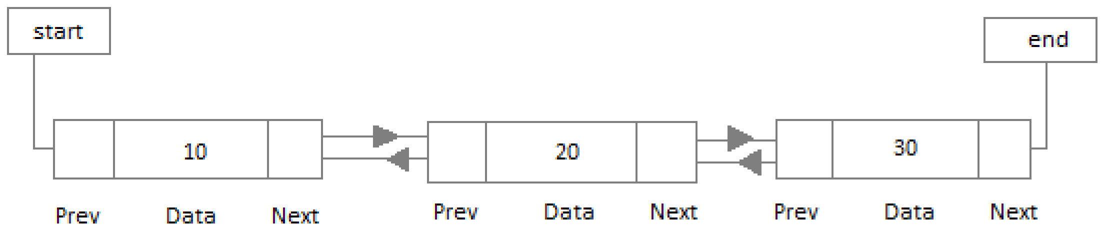
- Linked lists are used to implement stacks, queues, graphs, etc.
- Linked lists let you insert elements at the beginning and end of the list.
- In Linked Lists we don't need to know the size in advance.

Types of Linked Lists

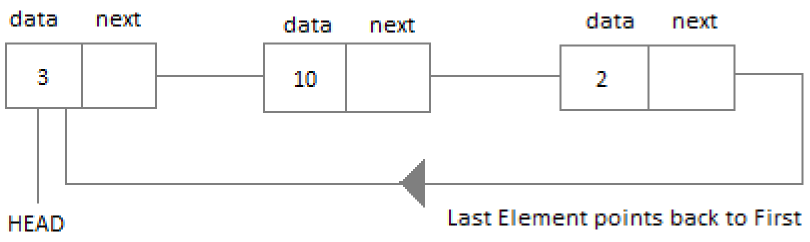
- **Singly Linked List :** Singly linked lists contain nodes which have a data part as well as an address part i.e. next, which points to the next node in sequence of nodes. The operations we can perform on singly linked lists are insertion, deletion and traversal.



- **Doubly Linked List :** In a doubly linked list, each node contains two links the first link points to the previous node and the next link points to the next node in the sequence.



- **Circular Linked List :** In the circular linked list the last node of the list contains the address of the first node and forms a circular chain.



Linear Linked List

The element can be inserted in linked list in 2 ways :

- Insertion at beginning of the list.
- Insertion at the end of the list.

We will also be adding some more useful methods like :

- Checking whether Linked List is empty or not.
- Searching any element in the Linked List
- Deleting a particular Node from the List

Before inserting the node in the list we will create a class **Node**. Like shown below :

```
class Node {
  public:
  int data;
  //pointer to the next node
  node* next;

  node() {
    data = 0;
    next = NULL;
  }

  node(int x) {
    data = x;
    next = NULL;
  }
}
```

We can also make the properties `data` and `next` as private, in that case we will need to add the getter and setter methods to access them. You can add the getters and setter like this :

```
int getData() {
  return data;
}

void setData(int x) {
  this.data = x;
}

node* getNext() {
  return next;
}

void setNext(node *n) {
  this.next = n;
}
```

Node class basically creates a node for the data which you enter to be included into Linked List. Once the node is created, we use various functions to fit in that node into the Linked List.

Linked List class

As we are following the complete OOPS methodology, hence we will create a separate class for **Linked List**, which will have all its methods. Following will be the Linked List class :

```
class LinkedList {
  public:
  node *head;
  //declaring the functions

  //function to add Node at front
  int addAtFront(node *n);
  //function to check whether Linked list is empty
  int isEmpty();
  //function to add Node at the End of list
  int addAtEnd(node *n);
  //function to search a value
  node* search(int k);
  //function to delete any Node
  node* deleteNode(int x);

  LinkedList() {
    head = NULL;
  }
}
```

Insertion at the Beginning

Steps to insert a Node at beginning :

1. The first Node is the Head for any Linked List.
2. When a new Linked List is instantiated, it just has the Head, which is Null.
3. Else, the Head holds the pointer to the first Node of the List.
4. When we want to add any Node at the front, we must make the head point to it.
5. And the Next pointer of the newly added Node, must point to the previous Head, whether it be NULL(in case of new List) or the pointer to the first Node of the List.
6. The previous Head Node is now the second Node of Linked List, because the new Node is added at the front.
7.

```
int LinkedList :: addAtFront(node *n) {
  int i = 0;
  //making the next of the new Node point to Head
  n->next = head;
  //making the new Node as Head
  head = n;
  i++;
  //returning the position where Node is added
  return i;
}
```

Inserting at the End

Steps to insert a Node at the end :

1. If the Linked List is empty then we simply, add the new Node as the Head of the Linked List.
2. If the Linked List is not empty then we find the last node, and make it' next to the new Node, hence making the new node the last Node.

```
int LinkedList :: addAtEnd(node *n) {

  //If list is empty

  if(head == NULL) {

    //making the new Node as Head

    head = n;

    //making the next pointe of the new Node as Null

    n->next = NULL;

  }

  else {

    //getting the last node

    node *n2 = getLastNode();

    n2->next = n;

  }

}


node* LinkedList :: getLastNode() {

  //creating a pointer pointing to Head

  node* ptr = head;

  //Iterating over the list till the node whose Next pointer points to null

  //Return that node, because that will be the last node.

  while(ptr->next!=NULL) {

    //if Next is not Null, take the pointer one step forward

    ptr = ptr->next;

  }

  return ptr;

}
```

Searching for an Element in the List

In searhing we do not have to do much, we just need to traverse like we did while getting the last node, in this case we will also compare the **data** of the Node. If we get the Node with the same data, we will return it, otherwise we will make our pointer point the next Node, and so on.

```
node* LinkedList :: search(int x) {

  node *ptr = head;

  while(ptr != NULL && ptr->data != x) {

    //until we reach the end or we find a Node with data x, we keep moving

    ptr = ptr->next;

  }

  return ptr;

}
```

Deleting a Node from the List

Deleting a node can be done in many ways, like we first search the Node with **data** which we want to delete and then we delete it. In our approach, we will define a method which will take the **data** to be deleted as argument, will use the search method to locate it and will then remove the Node from the List.

To remove any Node from the list, we need to do the following :

- If the Node to be deleted is the first node, then simply set the Next pointer of the Head to point to the next element from the Node to be deleted.
- If the Node is in the middle somewhere, then find the Node before it, and make the Node before it point to the Node next to it.

```
node* LinkedList :: deleteNode(int x) {
  //searching the Node with data x
  node *n = search(x);
  node *ptr = head;
  if(ptr == n) {
    ptr->next = n->next;
    return n;
  }
  else {
    while(ptr->next != n) {
      ptr = ptr->next;
    }
    ptr->next = n->next;
    return n;
  }
}
```

Checking whether the List is empty or not

We just need to check whether the **Head** of the List is `NULL` or not.

```
int LinkedList :: isEmpty() {
  if(head == NULL) {
    return 1;
  }
  else { return 0; }
}
```

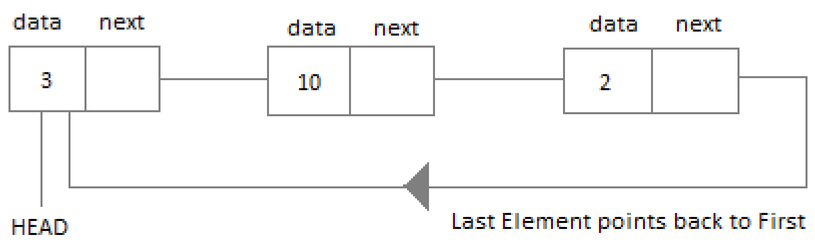
Now you know a lot about how to handle List, how to traverse it, how to search an element. You can yourself try to write new methods around the List.
If you are still figuring out, how to call all these methods, then below is how your `main()` method will look like. As we have followed OOP standards, we will create the objects of **LinkedList** class to initialize our List and then we will create objects of **Node** class whenever we want to add any new node to the List.

```
int main() {
  LinkedList L;
  //We will ask value from user, read the value and add the value to our Node
  int x;
  cout << "Please enter an integer value : ";
  cin >> x;
  Node *n1;
  //Creating a new node with data as x
  n1 = new Node(x);
  //Adding the node to the list
  L.addAtFront(n1);
}
```

Similarly you can call any of the functions of the LinkedList class, add as many Nodes you want to your List.

Circular Linked List

Circular Linked List is little more complicated linked data structure. In the circular linked list we can insert elements anywhere in the list whereas in the array we cannot insert element anywhere in the list because it is in the contiguous memory. In the circular linked list the previous element stores the address of the next element and the last element stores the address of the starting element. The elements points to each other in a circular way which forms a circular chain. The circular linked list has a dynamic size which means the memory can be allocated when it is required.



Application of Circular Linked List

- The real life application where the circular linked list is used is our Personal Computers, where multiple applications are running. All the running applications are kept in a circular linked list and the OS gives a fixed time slot to all for running. The Operating System keeps on iterating over the linked list until all the applications are completed.
- Another example can be Multiplayer games. All the Players are kept in a Circular Linked List and the pointer keeps on moving forward as a player's chance ends.

- Circular Linked List can also be used to create Circular Queue. In a Queue we have to keep two pointers, FRONT and REAR in memory all the time, where as in Circular Linked List, only one pointer is required.

Implementing Circular Linked List

Implementing a circular linked list is very easy and almost similar to linear linked list implementation, with the only difference being that, in circular linked list the last **Node** will have it's **next** point to the **Head** of the List. In Linear linked list the last Node simply holds NULL in it's next pointer.

So this will be oue Node class, as we have already studied in the lesson, it will be used to form the List.

```
class Node {
  public:
  int data;
  //pointer to the next node
  node* next;

  node() {
    data = 0;
    next = NULL;
  }

  node(int x) {
    data = x;
    next = NULL;
  }
}
```

Circular Linked List

Circular Linked List class will be almost same as the Linked List class that we studied in the previous lesson, with a few difference in the implementation of class methods.

```
class CircularLinkedList {
  public:
  node *head;
  //declaring the functions

  //function to add Node at front
  int addAtFront(node *n);
  //function to check whether Linked List is empty
  int isEmpty();
  //function to add Node at the End of list
  int addAtEnd(node *n);
  //function to search a value
  node* search(int k);
  //function to delete any Node
  node* deleteNode(int x);

  CircularLinkedList() {
    head = NULL;
  }
}
```

Insertion at the Beginning

Steps to insert a Node at beginning :

1. The first Node is the Head for any Linked List.
2. When a new Linked List is instantiated, it just has the Head, which is Null.
3. Else, the Head holds the pointer to the fisrt Node of the List.
4. When we want to add any Node at the front, we must make the head point to it.
5. And the Next pointer of the newly added Node, must point to the previous Head, whether it be NULL(in case of new List) or the pointer to the first Node of the List.
6. The previous Head Node is now the second Node of Linked List, because the new Node is added at the front.

```
int CircularLinkedList :: addAtFront(node *n) {
  int i = 0;
  /* If the list is empty */
  if(head == NULL) {
    n->next = head;
    //making the new Node as Head
    head = n;
    i++;
  }
  else {
    n->next = head;
    //get the Last Node and make its next point to new Node
    Node* last = getLastNode();
    last->next = n;
    //also make the head point to the new first Node
    head = n;
    i++;
  }
  //returning the position where Node is added
  return i;
}
```

Insertion at the End

Steps to insert a Node at the end :

1. If the Linked List is empty then we simply, add the new Node as the Head of the Linked List.
2. If the Linked List is not empty then we find the last node, and make it' next to the new Node, and make the next of the Newly added Node point to the Head of the List.

```
int CircularLinkedList :: addAtEnd(node *n) {
  //If list is empty
  if(head == NULL) {
    //making the new Node as Head
    head = n;
    //making the next pointer of the new Node as Null
    n->next = NULL;
  }
  else {
    //getting the last node
    node *last = getLastNode();
    last->next = n;
    //making the next pointer of new node point to head
    n->next = head;
  }
}
```

Searching for an Element in the List

In searhing we do not have to do much, we just need to traverse like we did while getting the last node, in this case we will also compare the **data** of the Node. If we get the Node with the same data, we will return it, otherwise we will make our pointer point the next Node, and so on.

```
node* CircularLinkedList :: search(int x) {
  node *ptr = head;
  while(ptr != NULL && ptr->data != x) {
    //until we reach the end or we find a Node with data x, we keep moving
    ptr = ptr->next;
  }
  return ptr;
}
```

Deleting a Node from the List

Deleting a node can be done in many ways, like we first search the Node with **data** which we want to delete and then we delete it. In our approach, we will define a method which will take the **data** to be deleted as argument, will use the search method to locate it and will then remove the Node from the List.

To remove any Node from the list, we need to do the following :

- If the Node to be deleted is the first node, then simply set the Next pointer of the Head to point to the next element from the Node to be deleted. And update the next pointer of the Last Node as well.
- If the Node is in the middle somewhere, then find the Node before it, and make the Node before it point to the Node next to it.
- If the Node is at the end, then remove it and make the new last node point to the head.

```
node* CircularLinkedList :: deleteNode(int x) {
  //searching the Node with data x
  node *n = search(x);
  node *ptr = head;
  if(ptr == NULL) {
    cout << "List is empty";
    return NULL;
  }
  else if(ptr == n) {
    ptr->next = n->next;
    return n;
  }
  else {
   while(ptr->next != n) {
      ptr = ptr->next;
    }
    ptr->next = n->next;
    return n;
  }
}
```

UNIT – 5

Graph & Tree algorithms

Graphs and Trees are at the heart of a computer and very useful in representing and resolving many real world problems like calculating the best paths between cities (Garmin, TomTom) and other famous ones like Facebook search and Google search engine crawling. If you know any programming language, you might have heard of garbage collection and for that the typical graph search called Breadth First Search is used. In this post, I will try to cover the following topics.

- Graphs – directed, undirected and special case of directed which are called Directed Acyclic Graph(DAG)
- Trees – Binary trees and also most useful version of it, the Binary Search Trees (BST)
- Standard traversal techniques – Breadth First Search (BFS), Depth First Search (DFS).

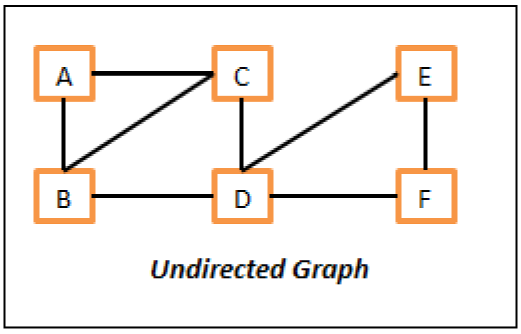
I know it will be a bit lengthy but I assure you it will be worth going through. I will try to cover the following topics in another post.

- Overview of shortest path algorithms – BFS, Dijkstra and Bellman Ford
- BST Search, Pre-order, In-order and Post-order.

Graphs

A graph consists of vertices (nodes) and edges. Think vertex as a point or place and the edge is the line/path connecting two vertices. In a typical graph and tree nodes there will be some kind of information is stored and the vertices will be associated with some cost. There are mainly two types of graphs and they are **undirected** and **directed** graphs.

In undirected graphs, all of the edges are undirected which means we can travel in either direction. So if we have an edge between vertex A and B, then we can travel from A to B and also from B to A. Following is the sample undirected graph.

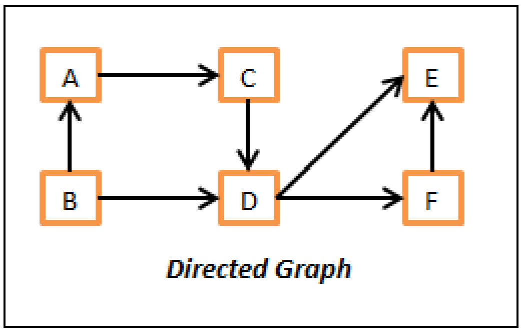


A Undirected Graph

Here in this undirected graph,

- The vertices are – A, B, C, D, E and F.
- The edges are – {A, B}, {A, C}, {B, C}, {B, D}, {C, D}, {D, E}, {D, F} and {E, F}. Here edge {A, B} means, we can move from A to B and also from B to A.

In directed graphs, all of the edges are directed which means we can only travel in the direction of an edge. So for example, if we have an edge between vertices A and B, then we can only go from A to B and not from B to A. If we need to go from B to A, then there should be another edge which should point B to A. Following is the sample directed graph.



Directed Graph

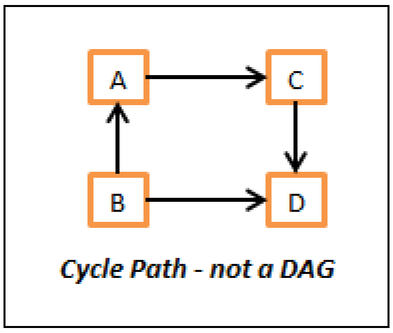
Here in this directed graph,

- The vertices are – A, B, C, D, E and F.
- The edges are – (A, C), (B, A), (B, D), (C, D), (D, E), (D, F) and (F, E). Here edge (A, C) means, we can only move from A to C and **not** from C to A.

The graphs can be mixed with both directed and undirected edges but let us keep this aspect away in this post to understand the core concepts better. Just an FYI the undirected edges are represented by curly braces '{}' where as directed edges are represented by '()'.

Directed Acyclic Graphs – DAG

This is a variation of standard directed graph without any cycle paths. It means there is no path where we can start from a vertex and comeback to the same starting vertex by following any path. By the way, **cycles in the graph** are very important and we **need to be very careful about** it. Otherwise, we could end up writing infinite algorithms which will never be completed.



Cycle in Graph

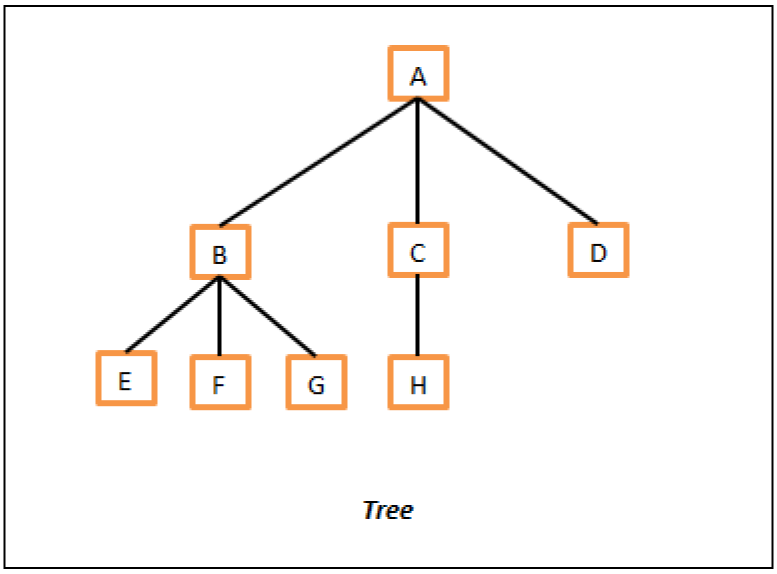
If we carefully look at the above listed directed graph, there is a cycle path and because of that, this graph is not a DAG.

Trees

This is another variation or we may call it subset of a graph. So we can say, all trees are graphs but not all graphs are trees. It is almost similar to the real world tree. It will have one top level vertex which is called root, and every node will have either zero or more child vertices. The best examples of tree data structure are family relationship and a company organization chart. Few important things to note down here,

- There will not be any cycles in this.
- Every node will have only one parent except the root which will not have any.

Following is a sample tree structure.

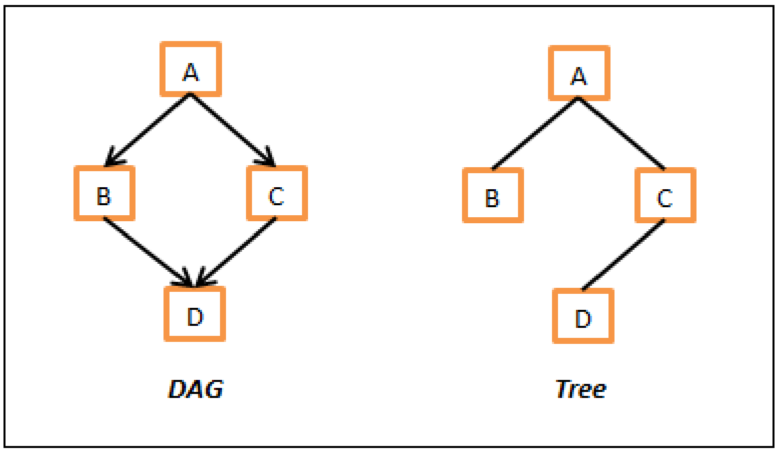


A Tree

As a standard the tree is represented from top to bottom. In this, the top node A is called as root of this tree as there are no parents to it. Then all of the child vertices (B, C and D) of A are represented on next level. All of the nodes which don't have any child nodes (E, F, G, H and D) are called leaf nodes.

DAG vs Tree

As you can see by now both the DAG and tree will not have any cycle paths but there is a key difference that is, in DAG there can be nodes with multiple parents but where as in tree, every node will have either a single parent or none. Following diagrams depicts this difference.

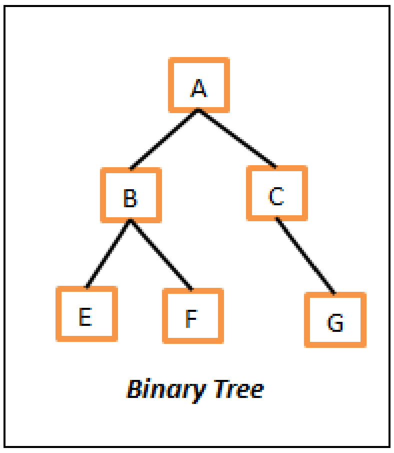


DAG vs Tree

In the above DAG example, D has two parents B and C and this is not possible in the tree. In tree example above, D has just one parent C.

Binary Tree

This is a version of the tree structure and in this every node can have strictly **at most two child nodes**. That means a node either can have two child nodes, just one or simply no child nodes in which case we call them leaf nodes. Following is a sample binary tree.



Binary Tree

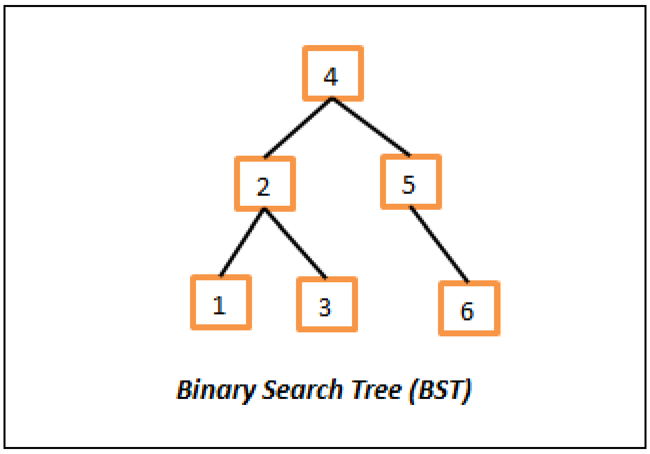
Binary Search Tree (BST)

This is a most useful version of the Binary Tree in which every node (including root) holds the following two properties.

- The **left child** node (if exists) should contain the value which is **less than or equal** to the parent.
- The **right child** node (if exists) should contain the value which is **greater than or equal** to the parent.

The above ordering property is really useful which we can understand clearly when we go through some of the search algorithms (especially the Binary search).

For simplicity, we will use the numbers in the nodes to show this relationship but the nodes can contain any information as long they are comparable. That means we should be able to compare them in some way and tell, whether those are equal or whether which one is greater. Following is a sample BST,



Binary Search Tree

Data Structure representation

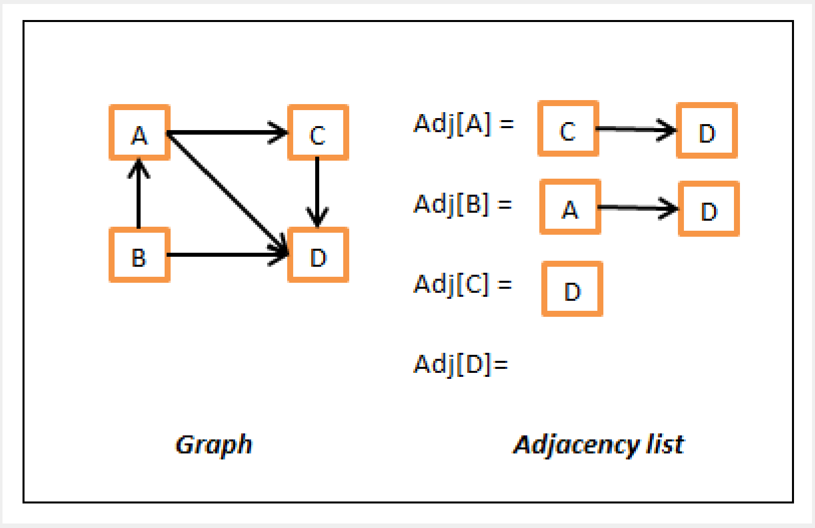
Hopefully all of the above details provide the insights on graphs and tree data structures. Before we get on to the actual traversals and searching algorithms, it is essential and useful to understand how they are represented or stored them in programming languages.

Following are the two mostly used representations.

Adjacency Lists

In this, the node is represented by an object and the edge is represented by the object references. Every object will have some data and a list of adjacent node references or a method which can return the list of adjacent nodes. If there are no neighbors, then it will be null or empty. So in nutshell, we have some mechanism to quickly find out neighbors for a given node.

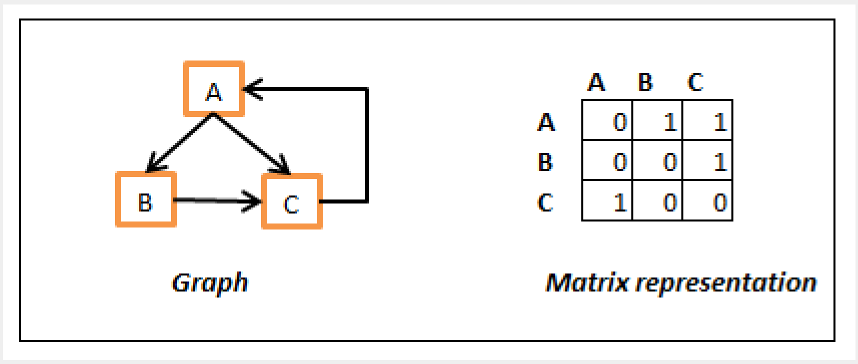
There is also another way to represent using adjacency lists which having a map which contains the key as the node and the value is list containing all of its neighbors. In this case, we can simply get all of its neighbors of a node N by simply looking for a key N in the map. That is we could say the neighbors of node N are simply Adj[N] where Adj is the adjacency map. Following is a sample representation of this representation.



Graph Adjacency list representation

Matrix representation

In this, we will use a double dimensional array where rows and columns represent the nodes and will use some kind of special value or actual edge cost as the value if they are connected. Following is sample example of representation.



	A	B	C
A	0	1	1
B	0	0	1
C	1	0	0

Graph matrix representation

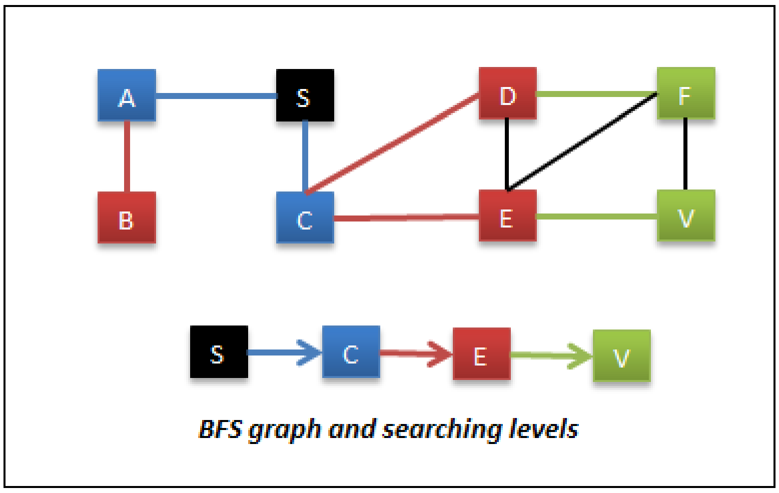
In the above, we have used 1 to represent the edge and 0 for no edge and we are looking from row to column. That is Array[B][C] is 1 as there is edge from B to C but Array[C][B] is zero as there is no edge between C to B. We could also use some value to represent the actual cost and use some sentinel (infinity or -1) value to represent no connection between the nodes. Yes, it is not a regulatory object oriented design but it have its own advantages like accessing random node and get the edge cost of a neighbor in constant time.

Searching Algorithms

There are two main search techniques which work fine on both the graphs and all versions of trees. It is just that we need to be careful about the cycles in graphs.

Breadth First Search (BFS)

In this, we will search or examine all of the sibling nodes first before going to its child nodes. In other words, we start from the source node (root node in trees) and visit all of the nodes reachable (neighboring nodes) from the source. Then we take each neighbor node and perform the same operation recursively until we reach destination node or till we run out of graph. The following color coded diagrams depicts this searching.



Breadth First Search

In this, the starting node is S and the target node is V. In this,

- Visit all of the nodes reachable from S. So we visit nodes A & C. These are the nodes reachable in 1 move or nodes at level 1.
- Then visit the nodes reachable from A & C. So we visit nodes B, D & E. These are the nodes reachable in 2 moves or we can say nodes at level 2.
- Then visit the nodes reachable from B, D & E. So we visit nodes F & V. These are the nodes reachable in 3 moves or we can say at level 3.

Advantages of this approach are,

- The path from which we first time reach a node can be considered as shortest path if there are no weights/costs to the edges or they might be constant. This is called single source shortest paths.
- This can be used in finding friends in social network sites like Facebook, Google+ and also linked in. In linked in, you might have seen the 1st, 2nd or 3rd degree as super scripts to the friend's names – it means that they are reachable in 1 move, 2 moves and 3 moves from you and now you might guess how they calculate that J

BFS implementation

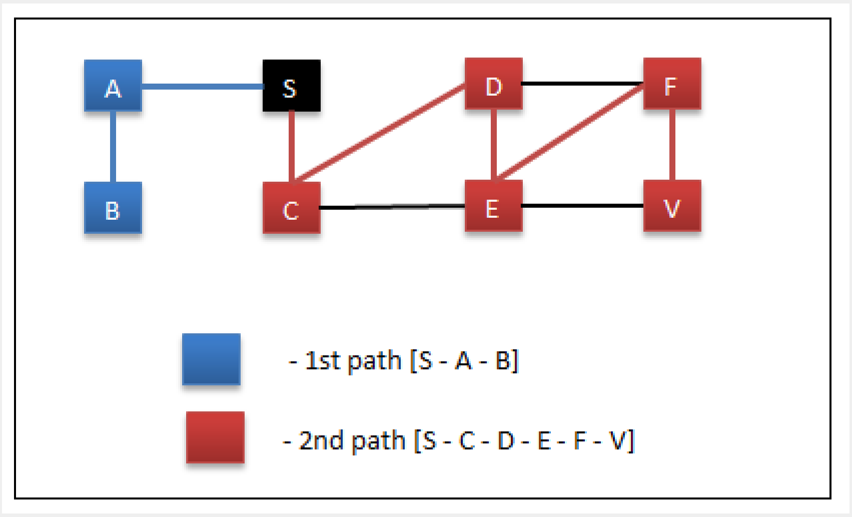
We can implement this in multiple ways. One of the most popular ways is by using Queue data structure and other by using lists to maintain the front and next tier nodes. Following is the java implementation of this algorithm.

```
public static void graphBFSByQueue(Node<String> source) {
  //if empty graph, then return.
  if (null == source) {
    return;
  }
  Queue<Node<String>> queue = new LinkedList<Node<String>>();
  //add source to queue.
  queue.add(source);
  visitNode(source);
  source.visited = true;
  while (!queue.isEmpty()) {
    Node<String> currentNode = queue.poll();
    //check if we reached out target node
    if (currentNode.equals(targetNode)) {
      return; // we have found our target node V.
    }
    //Add all of unvisited neighbors to the queue. We add only unvisited nodes to avoic
cycles.
    for (Node<String> neighbor : currentNode.neighbors) {
      if (!neighbor.visited) {
        visitNode(neighbor);
        neighbor.visited = true; //mark it as visited
        queue.add(neighbor);
      }
    }
  }
}

public static void graphBFSByLevelList(Node<String> source) {
  //if empty graph, then return.
  if (null == source) {
    return;
  }
  Set<Node<String>> frontier = new HashSet<Node<String>>();
  //this will be useful to identify what we visited so far and also its level
  //if we dont need level, we could just use a Set or List
  HashMap<Node<String>, Integer> level = new HashMap<Node<String>, Integer>();
  int moves = 0;
  //add source to frontier.
  frontier.add(source);
  visitNode(source);
  level.put(source, moves);
  while (!frontier.isEmpty()) {
    Set<Node<String>> next = new HashSet<Node<String>>();
    for (Node<String> parent : frontier) {
      for (Node<String> neighbor : parent.neighbors) {
        if (!level.containsKey(neighbor)) {
          visitNode(neighbor);
          level.put(neighbor, moves);
          next.add(neighbor);
        }
        //check if we reached out target node
        if (neighbor.equals(targetNode)) {
          return; // we have found our target node V.
        }
      }//inner for
    }//outer for
    moves++;
    frontier = next;
  }//while
}

public static void treeBFSByQueue(Node<String> root) {
  //if empty graph, then return.
  if (null == root) {
    return;
  }
  Queue<Node<String>> queue = new LinkedList<Node<String>>();
  //add root to queue.
  queue.add(root);
  while (!queue.isEmpty()) {
    Node<String> currentNode = queue.poll();
    visitNode(currentNode);
    //check if we reached out target node
    if (currentNode.equals(targetTreeNode)) {
      return; // we have found our target node V.
    }
    //Add all of unvisited neighbors to the queue. We add only unvisited nodes to avoic
```

```
cycles.
    for (Node<String> neighbor : currentNode.neighbors) {
      queue.add(neighbor);
    }
  }
}

public static void treeBFSByLevelList(Node<String> root) {
  //if empty graph, then return.
  if (null == root) {
    return;
  }
  List<Node<String>> frontier = new ArrayList<Node<String>>();
  //add root to frontier.
  frontier.add(root);
  while (!frontier.isEmpty()) {
    List<Node<String>> next = new ArrayList<Node<String>>();
    for (Node<String> parent : frontier) {
      visitNode(parent);
      //check if we reached out target node
      if (parent.equals(targetTreeNode)) {
        return; // we have found our target node V.
      }

      for (Node<String> neighbor : parent.neighbors) {
        next.add(neighbor);
      }//inner for
    }//outer for
    frontier = next;
  }//while
}
```

Depth First Search (DFS)

In this, we will search a node and all of its child nodes before proceeding to its siblings. This is similar to the maze searching. We search a full depth of the path and if we don't find target path, we backtrack one level at time and search all other available sub paths.



Depth First Search

In this,

- Start at node S.
  - Visit A
  - Visit B
  - Backtrack to A as B don't have any child nodes

- Backtrack to S as A don’t have any unvisited child nodes
- Backtrack to node S
    - Visit C
    - Visit D
    - Visit E
    - Visit F
    - Visit V – reached searching node so exit.

The path to node V from S is “S – C – D – E – F – V” and the length of it is 5. In the same graph, using BFS the path we found was “S – C – E – V” and the length of it is 3. So based on this observation, we could say DFS finds a path but it may or may not be a shorted path from source to target node.

Few of advantages of this approach are,

- The fastest way to find a path – it may not be a shortest one.
- If the probability of target node exists in the bottom levels. In an organization chart, if we plan to search a less experienced person (hopefully he will be in bottom layers), the DFS will find him faster compared to BFS because it explores the child nodes lot earlier compared to BFS.

DFS implementation

We can implement this in multiple ways. One is using the recursion and other is using Stack data structure. Following is the java implementation of this algorithm.

```
public static void graphDFSByRecersion(Node<String> currentNode) {
  if (null == currentNode) {
    return; // back track
  }
  visitNode(currentNode);
  currentNode.visited = true;
  //check if we reached out target node
  if (currentNode.equals(targetNode)) {
    return; // we have found our target node V.
  }
  //recursively visit all of unvisited neighbors
  for (Node<String> neighbor : currentNode.neighbors) {
    if (!neighbor.visited) {
      graphDFSByRecersion(neighbor);
    }
  }
}

public static void graphDFSByStack(Node<String> source) {
  //if empty graph, return
  if (null == source) {
    return; //
  }
  Stack<Node<String>> stack = new Stack<Node<String>>();
  //add source to stack
  stack.push(source);
  while (!stack.isEmpty()) {
    Node<String> currentNode = stack.pop();
    visitNode(currentNode);
    currentNode.visited = true;
    //check if we reached out target node
    if (currentNode.equals(targetTreeNode)) {
      return; // we have found our target node V.
    }
    //add all of unvisited nodes to stack
    for (Node<String> neighbor : currentNode.neighbors) {
      if (!neighbor.visited) {
        stack.push(neighbor);
      }
```

```
            }
        }
    }

    public static void treeDFSByRecersion(Node<String> currentNode) {
        if (null == currentNode) {
            return; // back track
        }
        visitNode(currentNode);
        //check if we reached out target node
        if (currentNode.equals(targetNode)) {
            return; // we have found our target node V.
        }
        //recursively visit all of unvisited neighbors
        for (Node<String> neighbor : currentNode.neighbors) {
                    graphDFSByRecersion(neighbor);
        }
    }

    public static void treeDFSByStack(Node<String> source) {
        //if empty graph, return
        if (null == source) {
            return; //
        }
        Stack<Node<String>> stack = new Stack<Node<String>>();
        //add source to stack
        stack.push(source);
        while (!stack.isEmpty()) {
            Node<String> currentNode = stack.pop();
            visitNode(currentNode);
            //check if we reached out target node
            if (currentNode.equals(targetTreeNode)) {
                return; // we have found our target node V.
            }
            //add all of unvisited nodes to stack
            for (Node<String> neighbor : currentNode.neighbors) {
                stack.push(neighbor);
            }
        }
    }
```